# Games and Puzzles

V. M. Sholapurkar
Department of Mathematics,
S. P. College, Pune

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No branch of intellectual activity is a more appropriate subject for discussion than puzzles and paradoxes... Puzzles in one sense, better than any single branch of mathematics reflect its always youthful, unspoiled and inquiring spirit

Kasner and Newman

# Distribution of Dollars

Bob has 10 pockets and 44 silver dollars. He wants his dollars so distributed into his pockets so that each pocket contains a different number of dollars. How Can he do that?

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Gauss proved this result at the age of 5 !!

# Page Numbers

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To number the pages of a bulky volume, the printer used 2989 digits. How many pages does the volume have?
Observe that for the printing of 9 pages, we need need 9 digits, 99 pages need $9 + 2(\times 90)$ digits. Similarly, 999 pages need $9 + 2(\times 90) + (3 \times 900) = 2889$ digits.
Now we have 2989 digits. So additional 100 digits have been used. But now we have four digit page nubers, resulting into 25 additional pages.
Thus we must have in all 999$+$ 25$=$ 1024 pages.

# Equal Sums

A farmer had 25 cows. He numbered them from 1 to 25. He would get as many litres of milk from each cow as it's number.. e.g one would get 5 litres milk from cow numbered 5.. The farmer had 5 sons. He wished to give five cows to each son so that each son would get equal amount of milk. How can he do so??

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| | | | | |
|---|---|---|---|---|
| 17 | 24 | 1 | 8 | 15 |
| 23 | 5 | 7 | 14 | 16 |
| 4 | 6 | 13 | 20 | 22 |
| 10 | 12 | 19 | 21 | 3 |
| 11 | 18 | 25 | 2 | 9 |

# Fake Note

A customer visited a shop and bought groceries worth Rs. 400 and handed over a note of Rs. 2000. As the shopkeeper did not have the change, he approached the shopkeeper next door and got the change. The shopkeeper then returned Rs. 1600 to the customer. In the afternoon, the shopkeeper next door brought the Rs. 2000 note and said, "The note you gave me this morning is fake." Convinced that the note was a counterfeit, the shopkeeper paid him Rs.2000 and tore the fake note of Rs.2000. So how much money did the shopkeeper lose in this whole transaction?

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The neighbouring shopkeeper has neither los nor profit.
So the loss of the shopkeeper is equal to the profit of the customer.
The customer got grocery worth Rs. 400 as well as cash of Rs. 1600. So customer's gain is Rs. 2000 which the shopkeeper lost.

# Game 1

Two players play a game that begins with player A calling out a number between 1 to 10. Player B then calls out another number; B's number must be bigger than A's number, but it cannot exceed A's number by more than 10. The players continue in this fashion. The winner of the game is the player who calls out the number 100. Which player can always win the game?

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Player A can always win the game by calling out numbers $1, 12, 23, 34, \cdots, 89, 100$.

# Vishamasur

There were 1000 people standing in a queue. A demon Vishamasur ate all people standing at odd positions in a queue (for example 1,3,5,7,9,...). Out of the remaining survivors, he again ate all people standing in odd positions. The process was repeated until there was only one survivor. Find the position of the survivor in the original queue of 1000 people.

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512 - the highest power of 2 less than 1000

Total 100 cards (numbered 1 to 100) were placed in a row all facing downwards. There were 100 people. The first person came and flipped all cards. Then the second person came and flipped all even number cards. After that the third person flipped all cards with numbers 3,6,9,12,... (which are multiples of 3). In general the k th person flipped all cards which are multiples of k. Finally which cards will remain in the position facing upwards?

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Now a bit of an experimentation would reveal that a number has odd number of divisors if and only if it is a perfect square.

# Wgudunni's Magic

The beautiful assistant of a magician placed a blindfold over the eyes of the famous stage magician. A member of the audience then rolled three dice. 'Multiply the number on the first dice by 2 and add 5,' said the magician. 'Then multiply the result by 5 and add the number on the second dice. Finally, multiply the result by 10 and add the number on the third dice.'

As he spoke, the assistant chalked up the sums on a blackboard which was turned to face the audience so that the magician could not have seen it, even if the blindfold had been transparent.

'What do you get?' the magician asked . 'Seven hundred and sixty-three,' said the assistant . Whodunni made strange passes in the air. 'Then the dice were—' What? And how did he do it?

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After carrying out the computations suggested by the magician, we get the number $10[5(2x + 5) + y] + z = 100x + 10y + 250 + z$ So after subtracting 250, we get the number $xyz$ in its decimal form.
So one can make out the digits $x, y$, and $z$.

# Game 2

Two players take turns putting coins on a round table one by one, without piling one coin on top of another. The player who cannot put the coin loses the game. Who will win?

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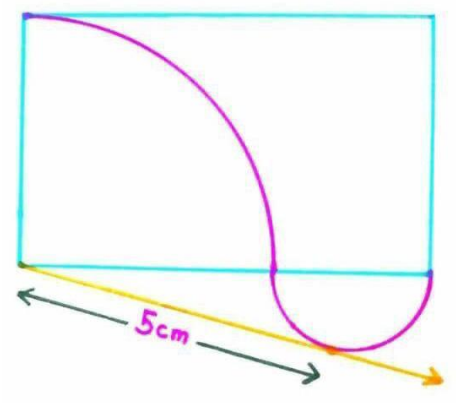
First player can win. He must place the first coin at the center of the table. After this he replies to each move of the second player by placing a coin in a symmetric position with respect to the center of the table.

In the following figure, find the area of the rectangle:



Let the larger side of the rectabgle be $y$ and the other side be $x$. Join the center of the lower semicircle to the point of contact the tangent line.
Then we get a right-angled triangle with sides 5 and $\frac{y-x}{2}$ and hypoteneous $\frac{y+x}{2}$. So the Pythagorus theorem gives the following equation:

$$25 + \frac{(y-x)^2}{4} = \frac{(y+x)^2}{4}$$

This gives $xy = 25$ i.e. the area of the rectangle is 25.

# Game 3

Ten 1's and ten 2's are written on blackboard. In one turn a player may erase any two figures. If the two figures erased are identical, they are replaced with a 2. If they are different, they are replaced with a 1. The first player wins if a 1 is left at the end and second player wins if a 2 is left. Who will win?

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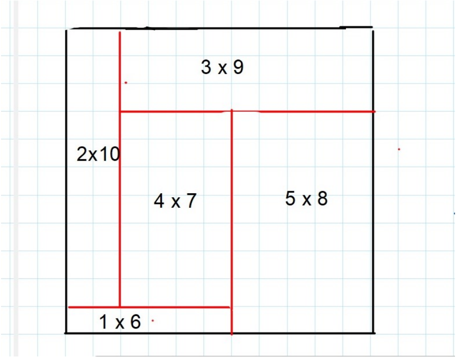
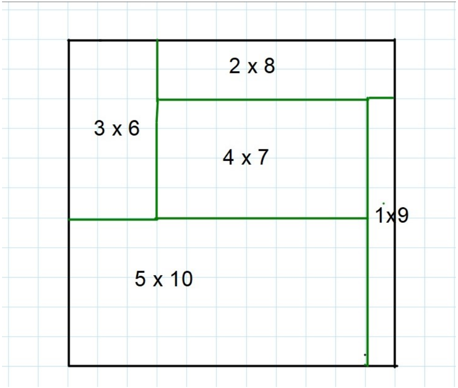
The parity of number of 1's on the blackboard remains unchanged after each move. Since there are evenly many 1's to begin with there cannot be a single 1 left at the end of the game. Therefore second player will win.

# Squaring the Rectangles

Form five rectangles by choosing their sides from the list 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, with each number being chosen exactly once. Then assemble the rectangles, without overlaps, to form an $11 \times 11$ square.


3 x 9
2x10
4 x 7
5 x 8
1 x 6




Friedrich Hartogs (1874-1943)

You have three jugs, which respectively hold 3 litres, 5 litres, and 8 litres of water. The 8-litre jug is full, the other two are empty. Your task is to divide the water into two parts, each of 4 litres, by pouring water from one jug into another. You are not allowed to estimate quantities by eye, so you can only stop pouring when one of the jugs involved becomes either full or empty.

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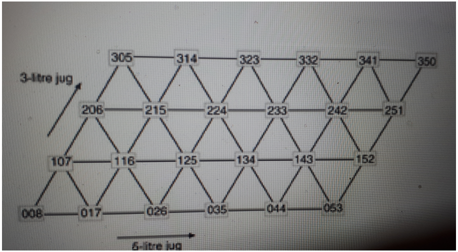
(8,0,0), (3,5,0), (3,2,3), (6,2,0), (6,0,2), (1,5,2), (1,4,3), (4,4,0)


3-litre jug
305
314
323
332
341
350
206
215
224
233
242
251
107
116
125
134
143
152
008
017
026
035
044
053
5-litre jug

M. C. K. Tweedie

# Crossing the Bridge

Four friends A, B, C and D need to cross a bridge. A maximum of two people can cross at a time. It is night and they have just 1 lamp. People that cross the bridge must carry the lamp to see the way. A pair must walk together at the speed of slower person. Speeds of A: 1 minute, B: 2 minutes, C: 7 minutes, D: 10 minutes to cross the bridge. What is the total minimum time required by all 4 friends to cross the bridge?

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17 minutes

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17 minutes
First pair 1 and 2 return 1
Second pair 10 and 7 return 2
Third pair again 1 and 2
Total: 17 minutes

Place each of the digits 1-8 in the eight circles, so that neighbouring digits (that is, those that differ by 1) do not lie in neighbouring circles (connected directly by a line).

## Avoiding Neighbours

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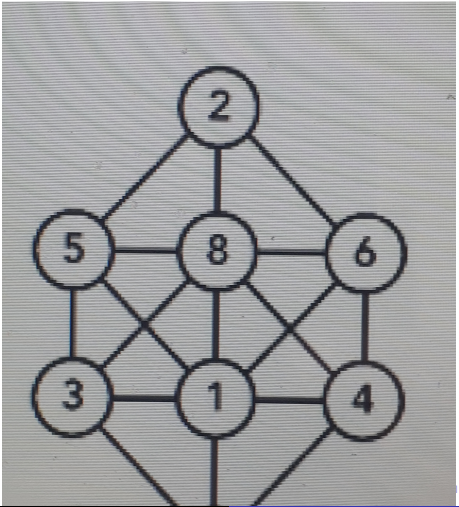

# Order into Chaos

There is a class of word puzzles in which you have to start with one word and turn it into a different one in such a way that only one letter is changed at each step and that every step is a valid word. Both words must have the same number of letters, of course. To avoid confusion, you are not allowed to rearrange the letters. So CATS can legitimately become BATS, but you can't go from CATS to CAST in one step. You can using more steps, though: CATS-CARS-CART-CAST. Here are two for you to try:
• Turn SHIP into DOCK. • Turn ORDER into CHAOS

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ORDER-OLDER-ELDER-EIDER-CIDER-CODER-CODES-CORESSORES-
SORTS-SOOTS-SPOTS-SHOTS-SHOPS-SHIPS-CHIPS-CHAPS- CHAOS

# References

- John H. Conway
- Henry Dudeney
- Martin Gradner
- Ian Stewart

# Thank You !